M.L.V. Textile & Engg. College, Bhilwara(Raj.)-311001

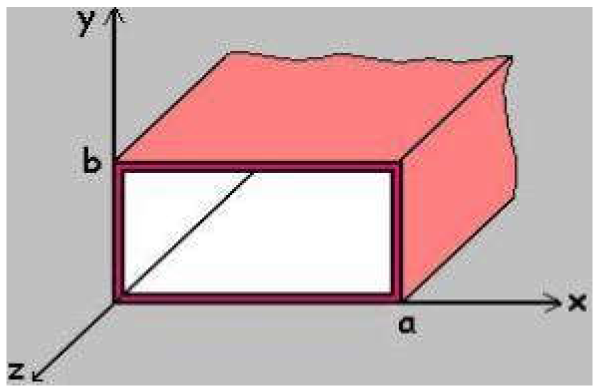
5ECE Mid term(2017-18) Subject:- ME Marks: 20

Q1. Explain TE_{mn} mode of operation in rectangular waveguide. Derive parameters of TE_{mn} mode. (5)

Ans.

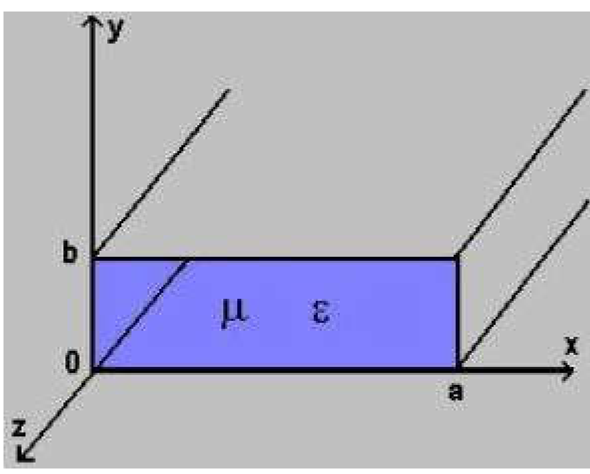
Rectangular waveguides are th one of the earliest type of the transmission lines. They are used in many applications. A lot of components such as isolators, detectors, attenuators, couplers and slotted lines are available for various standard waveguide bands between 1 GHz to above 220 GHz.

A rectangular waveguide supports TM and TE modes but not TEM waves because we cannot define a unique voltage since there is only one conductor in a rectangular waveguide. The shape of a rectangular waveguide is as shown below. A material with permittivity e and permeability m fills the inside of the conductor.



A rectangular waveguide cannot propagate below some certain frequency. This frequency is called the cut-off frequency.

TE Modes



Consider again the rectangular waveguide below with dimensions a and b (assume a>b) and the parameters **e** and **m.**

For TE waves $E_z = 0$ and H_z should be solved from equation for TE mode;

$$\tilde{N}^2_{xy} H_z + h^2 H_z = 0$$

Since $H_z(x,y,z) = H_z^0(x,y)e^{-gz}$, we get the following equation,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right)H_z^0(x,y) = 0$$

If we use the method of separation of variables, that is $H_z^0(x,y)=X(x).Y(y)$ we get,

$$-\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} = \frac{1}{Y(y)}\frac{d^2Y(y)}{dy^2} + h^2$$

Since the right side contains x terms only and the left side contains y terms only, they are both equal to a constant. Calling that constant as k_x^2, we get;

$$\frac{d^2X(x)}{dx^2} + k_x^2 X(x) = 0$$

$$\frac{d^2Y(y)}{dy^2} + k_y^2 Y(y) = 0$$

where $k_y^2=h^2-k_x^2$

Here, we must solve for X and Y from the preceding equations. Also we have the following boundary conditions:

$$\frac{\partial H_z^0}{\partial x} = 0 (E_y = 0) \quad \text{at } x=0$$

$$\frac{\partial H_z^0}{\partial x} = 0 (E_y = 0) \quad \text{at } x=a$$

$$\frac{\partial H_z^0}{\partial y} = 0 (E_x = 0)$$ at y=0

$$\frac{\partial H_z^0}{\partial y} = 0 (E_x = 0)$$ at y=b

From all these, we get

$$H_z^0(x,y) = H_0 \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$ (A/m)

From $k_y^2 = h^2 - k_x^2$, we have;

$$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

For TE waves, we have

$$H_x^0 = -\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial x}$$

$$H_y^0 = -\frac{\gamma}{h^2}\frac{\partial H_z^0}{\partial y}$$

$$E_x^0 = -\frac{jw\mu}{h^2}\frac{\partial H_z^0}{\partial y}$$

$$E_y^0 = -\frac{jw\mu}{h^2}\frac{\partial H_z^0}{\partial x}$$

From these equations, we obtain

$$E_x^0(x,y) = \frac{jw\mu}{h^2}\left(\frac{n\pi}{b}\right)H_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

$$E_y^0(x,y) = -\frac{jw\mu}{h^2}\left(\frac{m\pi}{a}\right)H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$H_x^0(x,y) = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$

$$H_y^0(x,y) = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)H_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)$$

where

$$\gamma = j\beta = j\sqrt{w^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

As explained before, m and n represent possible modes and it is shown as the TE_{mn} mode. m denotes the number of half cycle variations of the fields in the x-direction and n denotes the number of half cycle variations of the fields in the y-direction.

Here, the cut-off wave number is

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

and therefore, $\beta=\sqrt{k^2-k_c^2}$

The cut-off frequency is at the point where **g** vanishes. Therefore,

$$\boxed{f_c=\frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}\ (Hz)}$$

Since l=u/f, we have the cut-off wavelength,

$$\boxed{\lambda_c=\frac{2}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}\ (m)}$$

At a given operating frequency f, only those frequencies, which have $f>f_c$ will propagate. The modes with $f<f_c$ will not propagate.

The mode with the lowest cut-off frequency is called the dominant mode. Since TE_{10} mode is the minimum possible mode that gives nonzero field expressions for rectangular waveguides, it is the dominant mode of a rectangular waveguide with a>b and so the dominant frequency is

$$(f_c)_{10}=\frac{1}{2a\sqrt{\mu\varepsilon}}\ (Hz)$$

The wave impedance is defined as the ratio of the transverse electric and magnetic fields. Therefore, we get from the expressions for E_x and H_y (see the equations above);

$$Z_{TE}=\frac{E_x}{H_y}=\frac{jw\mu}{\gamma}=\frac{jw\mu}{j\beta}\Rightarrow Z_{TE}=\frac{k\eta}{\beta}$$

The guide wavelength is defined as the distance between two equal phase planes along the waveguide and it is equal to

$$\boxed{\lambda_g=\frac{2\pi}{\beta}}>\frac{2\pi}{k}=\lambda$$

which is thus greater than **l**, the wavelength of a plane wave in the filling medium.

The phase velocity is

$$u_p=\frac{w}{\beta}>\frac{w}{k}=\frac{1}{\sqrt{\mu\varepsilon}}$$

which is greater than the speed of the plane wave in the filling material.

The attenuation constant due to the losses in the dielectric is obtained as follows:

$$\gamma=j\beta=j\sqrt{k^2-k_c^2}=jk\sqrt{1-\left(\frac{f_c}{f}\right)^2}=jw\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}=jw\sqrt{\mu}\sqrt{\varepsilon+\frac{\sigma}{jw}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

After some manipulation, we get

$$\alpha_d=\frac{\sigma\eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}}=\frac{k^2\tan\delta}{2\beta}$$

Q2. Define microwave frequency. Write microwave frequency spectrum. (5)

Ans.

Microwaves are a form of electromagnetic radiation with wavelengths ranging from one meter to one millimeter; with frequencies between 300 MHz (100 cm) and 300 GHz (0.1 cm). Different sources define different frequency ranges as microwaves; the above broad definition includes both UHF and EHF (millimeter wave) bands.

Following are the applications of microwave engineering-
1. Antenna gain is proportional to the electrical size of the antenna. At higher frequencies, more antenna gain is therefore possible for a given physical antenna size, which has important consequences for implementing miniaturized microwave systems.
2. More bandwidth can be realized at higher frequencies. Bandwidth is critically important because available frequency bands in the electromagnetic spectrum are being rapidly depleted.
3. Microwave signals travel by line of sight are not bent by the ionosphere as are lower frequency signals and thus satellite and terrestrial communication links with very high capacities are possible.

Microwave frequency spectrum.

Band	Frequency Range (GHz)
L	1 to 2
S	2 to 4
C	4 to 8
X	8 to 10
Ku	12 to 18
K	18 to 26.5
Ka	26.5 to 40
Q	30 to 50
U	40 to 60
V	50 to 75
E	60 to 90 (millimeter waves)
W	75 to 110
F	90 to 140
D	110 to 170

Q3. An air-filled rectangular waveguide of dimension 7*3.5cm operates in the dominant TE_{10} mode. Find
a) Cut-off frequency, b) Determine phase velocity in guide at frequency 3.5 GHz, c) Determine the guided wavelength at same frequency.

Ans.

$$\text{a. } f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 7 \times 10^{-2}} = 2.14 \text{ GHz}$$

$$\text{b. } v_g = \frac{c}{\sqrt{1 - (f_c/f)^2}} = \frac{3 \times 10^8}{\sqrt{1 - (2.14/3.5)^2}} = 3.78 \times 10^8 \text{ m/s}$$

$$\text{c. } \lambda_g = \frac{\lambda_0}{\sqrt{1 - (f_c/f)^2}} = \frac{3 \times 10^8/(3.5 \times 10^9)}{\sqrt{1 - (2.14/3.5)^2}} = 10.8 \text{ cm}$$

Q4. What do you mean by scattering matrix? Explain how it is different from admittance and impedance matrix.

Ans. In a microwave junction there is an interaction of three or more components.There will be an output port, in addition there may be reflection from the junction of other ports. Totally there may be many combination, these are represented easily using a matrix called S matrix. Matrix is used in MW analysis to overcome the problem which occurs when H,Y & Z parameter are used in high frequencies.

Properties of s- matrix
1. it possess symmetric properties sij=sji
2. it possess unitary property
3. [s][s]*=[i]